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STATISTICAL SUBSPACE BASED DAMAGE LOCALIZATION ON SAINT-NAZAIRE BRIDGE MOCK-UP

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ABSTRACT
The subject of damage localization is an important issue for Structural Health Monitoring (SHM) particularly in mechanical or civil structures under ambient excitation. In this paper, the statistical subspace-based damage localization method has been applied on a benchmark application, namely a 1/200 scale model of the Saint-Nazaire Bridge, which is a cable-stayed bridge located on the Loire River near the river's mouth. The employed damage localization method combines data-driven features with physical parameter information from a finite element model in statistical tests, avoiding typical ill-conditioning problems of FE model updating. Damage is introduced in the mockup for cable failures on some of the 72 cables. The purpose of the experiment is to assess the capability of damage assessment methods to find a cable failure.

Keywords: damage localization, cable-stayed bridge, cable failure, structural health monitoring.

1. INTRODUCTION

Vibration-based structural health monitoring has become an important issue during the last decades [1–3], especially for bridges, buildings or offshore structures. We consider the case of output-only vibration measurements of a benchmark structure subject to ambient excitation.

Methods for damage detection are widely developed since they can operate purely data-based and do not require a finite element (FE) model of the monitored structure. Automated methods for damage localization are more sophisticated since a link between the measurement data and the physical properties of the structure is required, which is often given by a FE model. Experimental works on complex structures are required to validate the methods. Experimental works are usually focused on the study of two categories of structures: models tested in the laboratory, or real structures damaged and tested before destruction.

A lab benchmark that has been evaluated by several research groups is e.g. the ASCE Benchmark Test Frame at the University of British Columbia in Vancouver, Canada, which was established in 2002 [4]. Recent ambient vibration tests have been performed on the structure in 2017 [5].

In previous work [6], a benchmark for damage diagnosis on a cable-stayed bridge structure was proposed using output-only vibration data. The experimental work was carried out in the lab on a model scale 1/200 of the bridge of Saint-Nazaire in France, equipped with 10 accelerometers. The introduced damage is the rupture of one or two of the cables supporting the deck (see some examples in [7]). In [6], the statistical stochastic damage locating vector approach (S-SDDLV) was successfully applied to localize the damage. In this paper, the statistical subspace based residual approach [8, 9] is applied for damage localization. It operates on a data-driven residual vector that is statistically evaluated using information from a FE model, combining both data-driven and model-based methodologies.

This paper is organized as follows. In Section 2., the modeling of the structure is recalled. In Section 3., the damage localization framework is detailed and finally, applied on the bridge mockup in Section 4..

2. MODELING OF THE STRUCTURE

The behavior of a mechanical structure is assumed to be described by a linear time-invariant (LTI) dynamic system

$$M\ddot{\mathcal{X}}(t) + C\dot{\mathcal{X}}(t) + K\mathcal{X}(t) = f(t) \tag{1}$$

where M, C, $K \in \mathbb{R}^{d\times d}$ are the mass, damping and stiffness matrices, respectively, t indicates continuous time and $\mathcal{X} \in \mathbb{R}^d$ denotes the displacements at the d degrees of freedom (DOF) of the structure. The external force $f(t)$ is not measurable and modeled as white noise. Let the dynamic system (1) be observed at r coordinates. Defining the state vector $x = [\mathcal{X} \;\; \dot{\mathcal{X}}]^T$ and sampling at time steps $t = k\tau$ yields the discrete-time state-space model

$$\left\{ \begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \right. , \tag{2}$$

with state vector $x_k \in \mathbb{R}^n$, output vector $y_k \in \mathbb{R}^r$, the state transition matrix $A \in \mathbb{R}^{n\times n}$ and output matrix $C \in \mathbb{R}^{r\times n}$, where $n = 2d$ is the system order and r is the number of outputs.

3. DAMAGE LOCALIZATION FRAMEWORK

Let $\theta \in \mathbb{R}^l$ be a parameter vector that describes the monitored system in the current state, and $\theta_0 \in \mathbb{R}^l$ its value in the healthy reference system. We assume that damage is linked to stiffness changes, thus θ is the collection of stiffness parameters of the elements of the structure, where θ_0 is obtained from a finite element model in the reference state. For example, the components of θ can be the stiffnesses of a system, Young modulus of beam elements or it can be basically any quantity linked to damage-sensitive properties of the system.

In [8], a statistical framework has been set up for Gaussian residual vectors parametrized by θ with the purpose to decide which parts of θ have changed for damage localization. The Gaussian residual vector $\zeta \in \mathbb{R}^h$ is computed from the measurements of the system and needs to satisfy

$$\zeta \sim \left\{ \begin{array}{ll} \mathcal{N}(0, \Sigma) & \text{in reference state} \\ \mathcal{N}(\mathcal{J}\delta, \Sigma) & \text{in damaged state,} \end{array} \right. \tag{3}$$

with $\delta = \sqrt{N}(\theta - \theta_0) \in \mathbb{R}^l$ is the unknown change in parameter vector, N is the data length used for the computation of ζ, the sensitivity matrix $\mathcal{J} \in \mathbb{R}^{h\times l}$ has full column rank and the residual covariance matrix $\Sigma \in \mathbb{R}^{h\times h}$ is positive definite.

3.1. Subspace-based residual

In previous works on damage detection and localization [8, 10], the subspace-based residual function

$$\zeta^s \stackrel{\text{def}}{=} \sqrt{N}\,\mathrm{vec}(S^T\widehat{\mathcal{H}}) \tag{4}$$

has been introduced, where $\widehat{\mathcal{H}}$ is an estimate of the block Hankel matrix of the output covariances of the current system (2)

$$\mathcal{H} \stackrel{\text{def}}{=} \begin{bmatrix} R_1 & R_2 & \dots & R_q \\ R_2 & R_3 & \dots & R_{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & R_{p+2} & \dots & R_{p+q} \end{bmatrix}, \quad R_i = \mathbf{E}(y_k y_{k-i}^T), \quad \hat{R}_i = \frac{1}{N}\sum_{k=1}^{N} y_k y_{k-i}^T$$

and S is the left null space of $\mathcal{H}$ from the reference system. It can be obtained from a Hankel matrix $\mathcal{H}_0$ in the reference state through a singular value decomposition

$$\mathcal{H}_0 = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \approx 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

as $S = U_2$. When the system is in the reference state, the expected value of the product $S^T\widehat{\mathcal{H}}$ is zero, and when the system is damaged the product deviates from zero. The residual in (4) satisfies relation (3) asymptotically, and its sensitivity and covariance are given in detail in [10].

In the following, the statistical tests for damage localization are recalled.

3.2. Damage localization tests

For damage localization it has to be decided which parts of vector δ in (3) are non-zero, i.e. which parts of the parameter vector are changed. The structural elements corresponding to the changed parameters are thus damaged. To this end, each component of δ will be tested one after another. Denote the component to be tested by δ_a, and the remaining components by δ_b, such that

$$\delta = \begin{bmatrix} \delta_a \\ \delta_b \end{bmatrix}. \tag{5}$$

Then, $\delta_a = 0$ is tested against $\delta_a \neq 0$. Following (5), the sensitivity matrix $\mathcal{J}$ and the Fisher information matrix $F = \mathcal{J}^T\Sigma^{-1}\mathcal{J}$ are analogously arranged as

$$\mathcal{J} = \begin{bmatrix} \mathcal{J}_a \; \mathcal{J}_b \end{bmatrix}, F = \begin{bmatrix} F_{aa} & F_{ab} \\ F_{ba} & F_{bb} \end{bmatrix} = \begin{bmatrix} \mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_a & \mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_b \\ \mathcal{J}_b^T\Sigma^{-1}\mathcal{J}_a & \mathcal{J}_b^T\Sigma^{-1}\mathcal{J}_b \end{bmatrix}. \tag{6}$$

Direct tests Assuming that $\delta_b = 0$ for testing $\delta_a = 0$ against $\delta_a \neq 0$, the generalized likelihood ratio (GLR) test follows as

$$t_{dir} = \zeta^T\Sigma^{-1}\mathcal{J}_a^T(\mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_a)^{-1}\mathcal{J}_a^T\Sigma^{-1}\zeta, \tag{7}$$

which is called direct test. The test statistic t_{dir} is χ^2 distributed with non-centrality parameter $\delta_a^T F_{aa}\delta_a$. For making decision about the damage location, the test variable is compared to a threshold. Note that the simplifying assumption $\delta_b = 0$ in this test that may lead to false alarms when testing an undamaged element while another element is actually damaged.

Minmax tests Instead of assuming the components of $\delta_b = 0$, the variable δ_b is substituted by its least favorable value for making a decision about δ_a, which leads to the minmax test as follows. Define the partial residuals

$$\zeta_a = \mathcal{J}_a^T\Sigma^{-1}\zeta \tag{8a}$$

$$\zeta_b = \mathcal{J}_b^T\Sigma^{-1}\zeta, \tag{8b}$$

and the robust residual

$$\zeta_a^* = \zeta_a - F_{aa}F_{bb}^{-1}\zeta_b,$$

whose mean is sensitive to changes δ_a but not to δ_b. Testing $\delta_a = 0$ against $\delta_a \neq 0$ with the GLR test yields

$$t_{mm} = \zeta_a^* F_a^{*-1} \zeta_a^*, \tag{9}$$

where $F_a^* = F_{aa} - F_{ab}F_{bb}^{-1}F_{ba}$. The test statistic t_{mm} is χ^2 distributed with non-centrality parameter $\delta_a^T F_a^* \delta_a$.

A prerequisite for the localization approach with the minmax tests is the independence of the parameterization in the sense that matrix (6) needs be full rank. This can be ensured through clustering the vectors of the normalized sensitivity matrix $\Sigma^{-1/2}\mathcal{J}$ and defining $\mathcal{J}_b$ as the cluster centers of the clusters not containing the currently tested element a. An adequate clustering approach for this problem has been shown to be complete-linkage clustering [9, 11].

4. APPLICATION: SAINT-NAZAIRE BRIDGE MOCK-UP

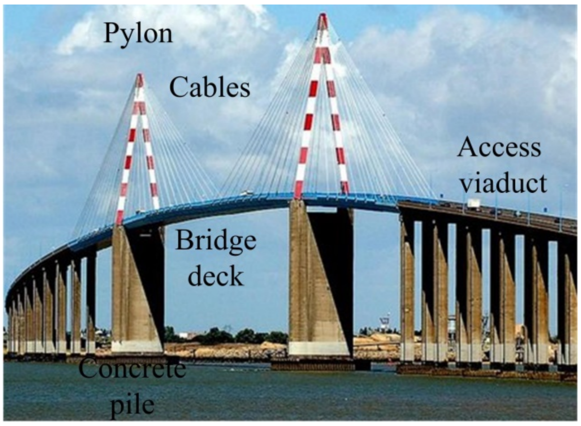
In this application, the statistical subspace method for damage localization is applied to the Saint-Nazaire bridge mockup. In previous work [6], the details of the experiment were presented and are shortly recalled here.

4.1. The real structure

The Saint-Nazaire Bridge is a cable-stayed bridge located in the Loire River near the river's mouth in the west of France. The bridge includes two access viaducts supported by pile foundations (Figure 1a). The northern viaduct (1115 m long) and the southern one (1521 m long) are made of pre-stressed concrete. The pile foundations of the central part are also made of concrete. The main structure (in blue on Figure 1a) is composed of a 720 m long cable-stayed metallic frame. The bridge deck, the cables, and the triangular bridge pylons are made of steel.

Some keys features are listed below to describe the structure:

- 72 cables
- 56 concrete piles
- Total length: 3356 meters
- Central structure: 720 meters
- Central span: 404 meters
- Height of deck of central bridge: 68 meters
- Height of the two central pylons: 130 meters
- Air draft of 61 meters



(a) View of the real structure

(b) Mockup of the central structure

Figure 1: Saint Nazaire bridge.

4.2. The mock-up and acquisition system

For the dynamic analysis of the structure, only the central bridge (metallic part) was considered. The study presented here was carried out on a 1/200 scale model of the structure, i.e. a 3.6 m long mockup positioned on two surface plates (Figure 1b). To obtain the geometrical and mechanical characteristics of the model, the length of the real structure is multiplied by a scale factor of 1/200. In order to preserve the scale factor for the masses, it was chosen to use building materials for the model with density similar or close to those used for the construction of the real bridge.

On the bridge model, there are 10 miniature piezoelectric accelerometers (0.8 gm) of sensitivity 100 mV/g (PCB of reference ICP) capable of measuring vertical accelerations. All the signals are collected on a data-logger (HBM reference MX16101) at the acquisition frequency of 4800 Hz. The excitation is provided by an audio boomer (see Figure 2a) which acts as a shaker. It is powered by the signal of a white noise generator (+/- 5V peak to peak, manufactured by Tektronix, reference AFG), amplified by a stereo amplifier. The signal is controlled with an oscilloscope. The devices of the signal acquisition chain and the shaker are visible in Figure 2a.

In the damaged and healthy states, acceleration data containing 581,118 and 588,620 samples are recorded, respectively.

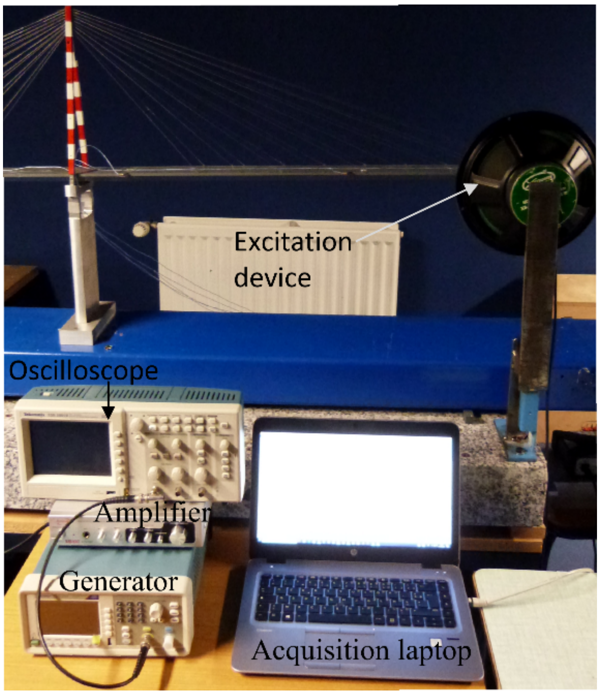
4.3. Damage to structure

The damage to assess is a failure of a cable supporting the deck. It is introduced by removing the fastener connecting the cable and the deck. Thus, only the stiffness of the structure is modified and not its mass. Two damages are tested, the rupture of a cable, and the rupture of two cables symmetrical with respect to the pylon. These two elements located in the middle of the bridge are visible in Figure 2b.

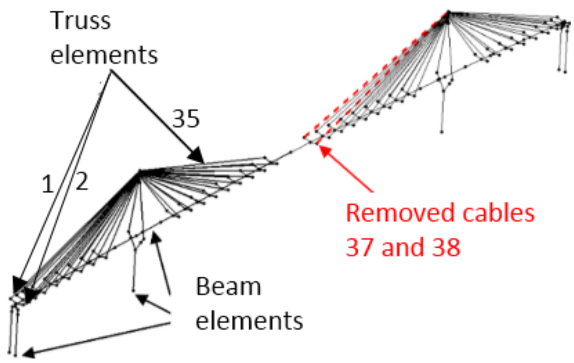
4.4. The Saint Nazaire bridge mock-up finite element model and parameterization

The FE model has been built (see Figure 2b) with the student version of the ABAQUS software. The model consists of 180 beam elements and 72 truss elements for a total of 1062 degrees of freedom. The damaged elements are numbered 37 and 38.

The system parameter θ is defined as the collection of cable stiffnesses of each of the 72 cables, since it is the goal to localize the damage in these cables. Using a finite difference method, the sensitivity of the mode shapes and natural frequencies with respect to each cable stiffness is computed for the first 15 vertical modes obtained from the FE model. To analyze the impact of modal truncation in the residual sensitivity on the localization approach, 10 and 15 modes are used in its computation, respectively.



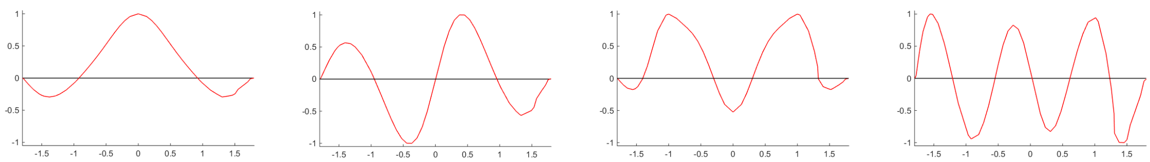
(a) Acquisition and excitation devices



(b) Finite element model

Figure 2: Saint Nazaire bridge mockup.

The first four vertical mode shapes are shown in Figures 3. The cluster dendrogram is shown in Figure 4, visualizing the cluster construction based on a statistical distance measure between the sensitivities of the different cables that will be tested for damage. This distance corresponds to the cosine of the angle between the sensitivities, varying between 0 and 1. If the sensitivities of two elements are close, they are put into the same cluster, and only the cluster center is used in the minmax test as detailed in Section 3.2.. A threshold of 0.15 is set for close elements to be in the same cluster, indicated by the same colors in Figure 4.



(a) 1^{st} mode at 35.9 Hz **(b)** 2^{nd} mode at 61.1 Hz **(c)** 3^{rd} mode at 88.6 Hz **(d)** 4^{th} mode at 121.5 Hz

Figure 3: Modeshapes (vertical) from FE model in the healthy state.

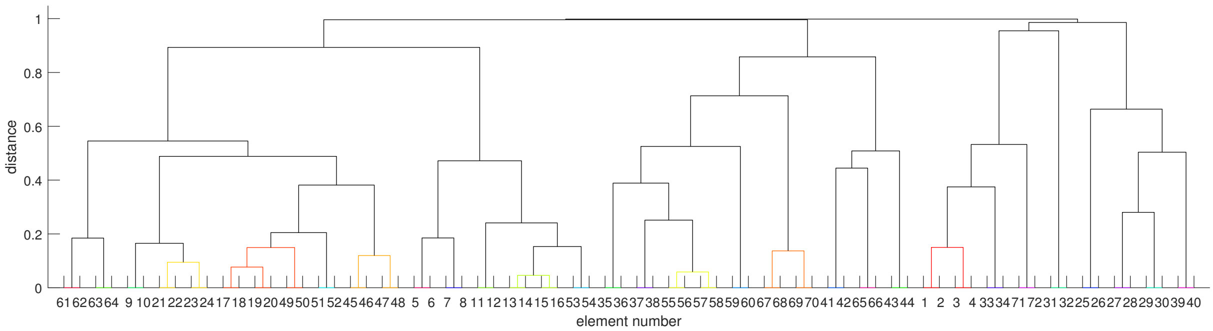
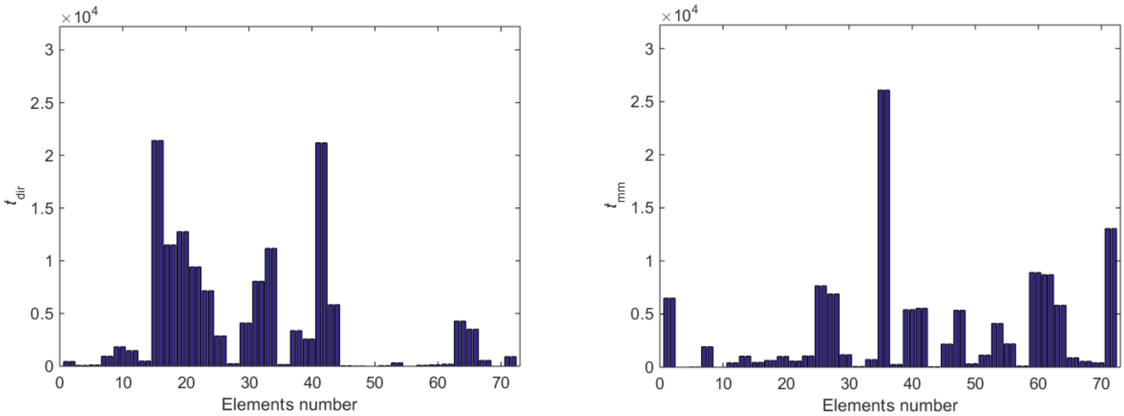


Figure 4: Dendrogram for clustering using 15 modes.

4.5. Localization results

The localization test statistics are computed for all 72 elements, which are corresponding to the cable stiffness (see Figure 2b), using a dataset in both damaged and healthy states. Note that damage was introduced by removing first cable number 37, and then cables 37 and 38 at the middle of the structure. The highest value of the test statistic indicates the most likely damage localization. The results are shown in Figures 5 - 6 and 7 - 8 using 10 and 15 vertical modes computed from the FE model, respectively, for both the direct and minmax tests, and the damage cases with one and two damaged cables.

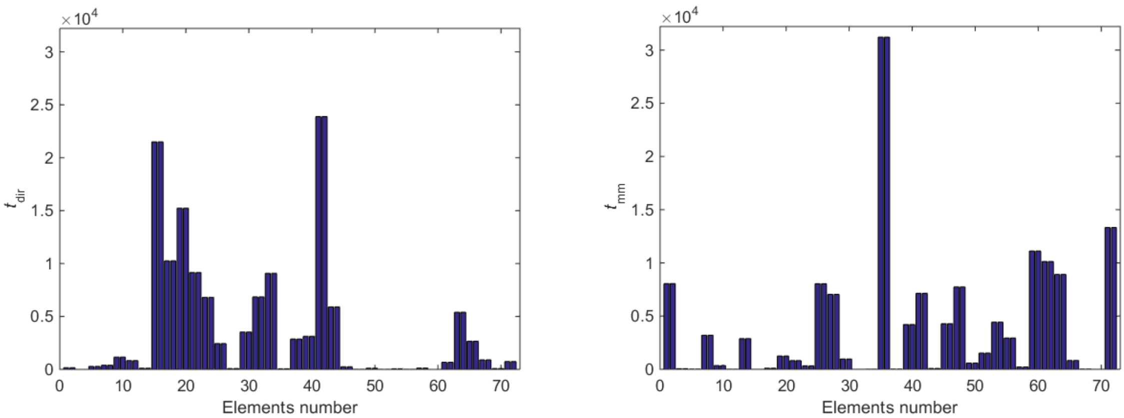
Using 10 vertical modes in the sensitivity computation, it can be seen that the damage is not correctly localized in the direct and minmax tests for the respective test cases in Figures 5 and 6. However, the direct and minmax tests have the highest test values for the neighboring cable of the damaged elements. While the test statistic for other elements also react strongly in the direct test, this is much less the case in the minmax test. The performance for the tests for one and two damaged cables in Figures 5 and 6 is comparable.



(a) Direct tests

(b) Minmax tests

Figure 5: Test case 1: Localization for damaged cable 37, using 10 modes.

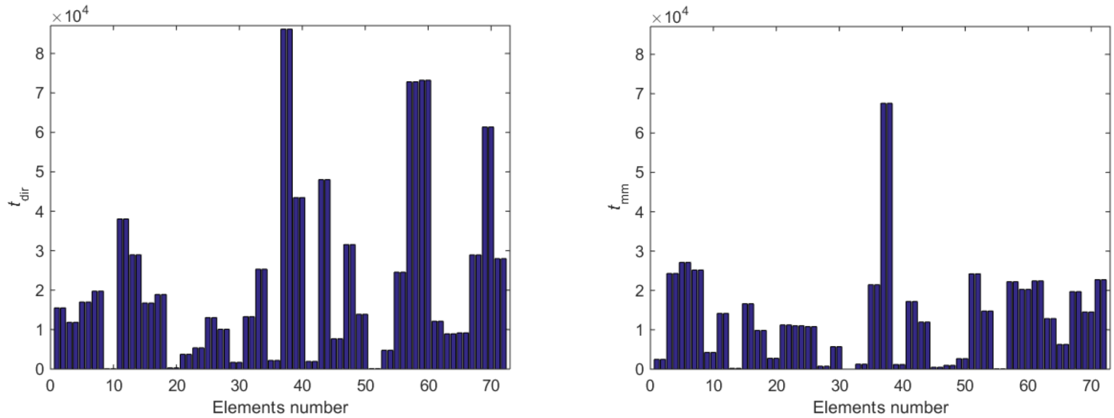


(c) Direct tests

(d) Minmax tests

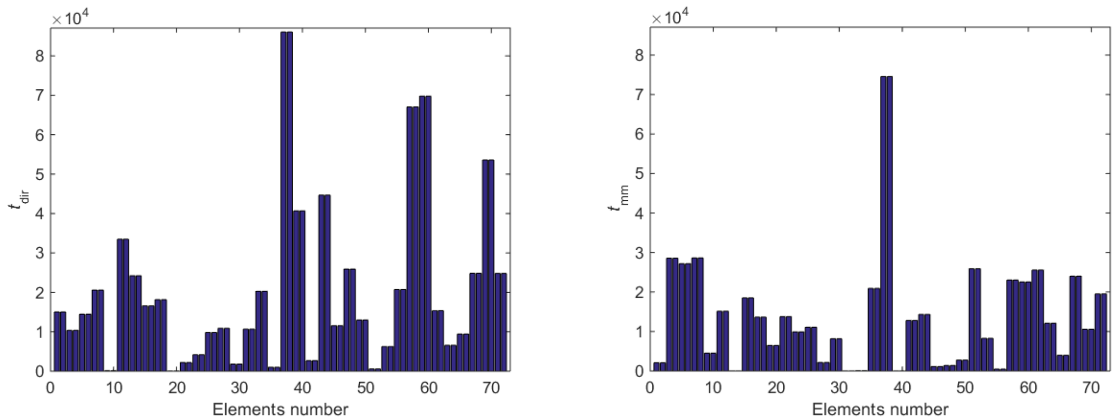
Figure 6: Test case 2: Localization for damaged cables 37 and 38, using 10 modes.

Considering 15 vertical modes instead of 10 in the sensitivity computation, it can be seen in Figures 7 and 8 that the highest value of the direct and minmax tests is correctly located in the damaged elements, which may be due to less modal truncation in the sensitivity computation. Note that the test for both elements 37 and 38 react in the same way, even if just one of them is damaged, since they cannot be distinguished by their sensitivities (see Figure 4). The direct test in Figures 7a and 7c reacts also at undamaged elements due to the violation of the assumptions in the test (see Section 3.2.), which clearly leads to false positive results in this case. In the minmax tests in Figures 7b and 7d, only the actual damaged elements show a strong test reaction, separating the damaged elements clearly from the undamaged ones.



(a) Direct tests **(b)** Minmax tests

Figure 7: Test case 1: Localization for damaged cable 37, using 15 modes.



(c) Direct tests **(d)** Minmax tests

Figure 8: Test case 2: Localization for damaged cables 37 and 38, using 15 modes.

5. CONCLUSIONS

In this paper, the statistical subspace based damage localization method has been applied on output-only vibration measurements of a benchmark structure, the Saint-Nazaire bridge mockup. Here, two test cases were considered with one and two cable failures. In both test cases, the introduced damage was correctly localized at the damaged elements when the number of considered modes computed from the FE model was sufficiently high. It is noted that the FE model was not updated, and the conditions on its precision with the used method are rather light. Future work should include damage quantification using this approach.

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